Infinite Impulse Response Filters Design
(UNIT-3)

"Ideal" FIR filters

- In general, an ideal (continuous) frequency response is related to an (infinite) impulse response by the Fourier Series

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-jn\omega}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{jn\omega} d\omega$$

- The coefficients of an "ideal" FIR filter can therefore be found from the Fourier Series coefficients of the desired frequency response.

- Not practical because
  - the impulse response cannot be infinite
  - the impulse response must be causal
  - maybe don't need the frequency response to be specified for all (continuous) values of ω

Frequency Sampling

- truncation of the impulse response introduces errors
  - truncation of the impulse response is equivalent to sampling of the frequency response
- the truncated impulse response can be obtained directly from the DFT of the desired frequency response

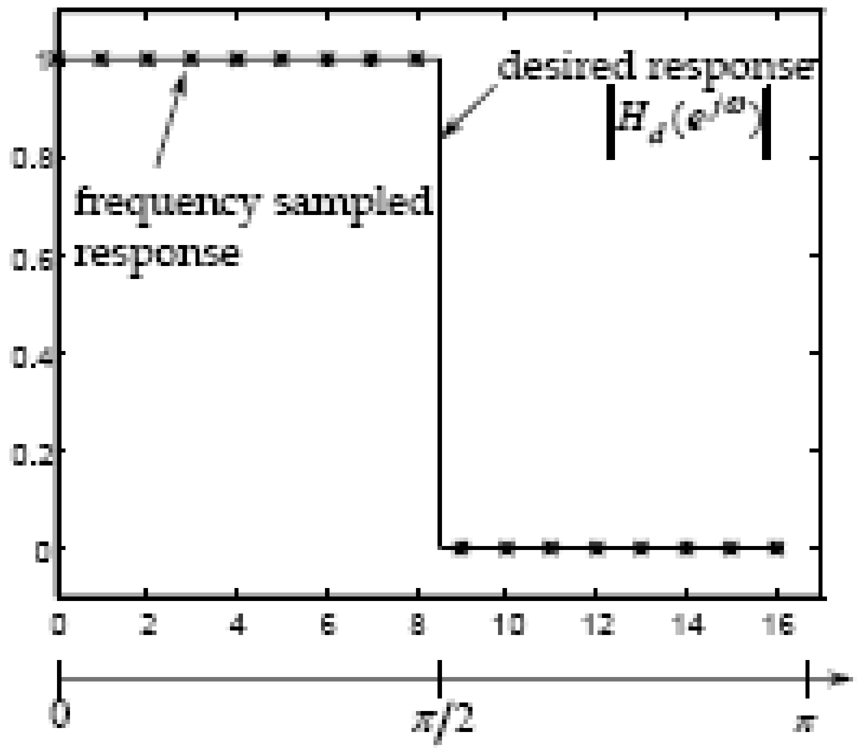
$$H(k) = H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi}{N}k} = \sum_{n=0}^{N-1} h(n) e^{-j\frac{2\pi}{N}nk} \qquad k = 0,1,2,\ldots,N-1$$

$$h(n) = \frac{1}{N}\sum_{k=0}^{N-1} H(k) e^{j\frac{2\pi}{N}nk} \qquad n = 0,1,2,\ldots,N-1$$

- N-1 is the order of the FIR filter
- The frequency response has been sampled at N points around the unit circle
  - The frequency response of filter designed in this way will only be exactly correct only at these points

- **Example**
  - Lowpass filter
  - Number of taps: 33

$$|H(e^{j\omega})| = \begin{cases} 1, & |\omega| < \pi/2 \\ 0, & \text{otherwise} \end{cases}$$

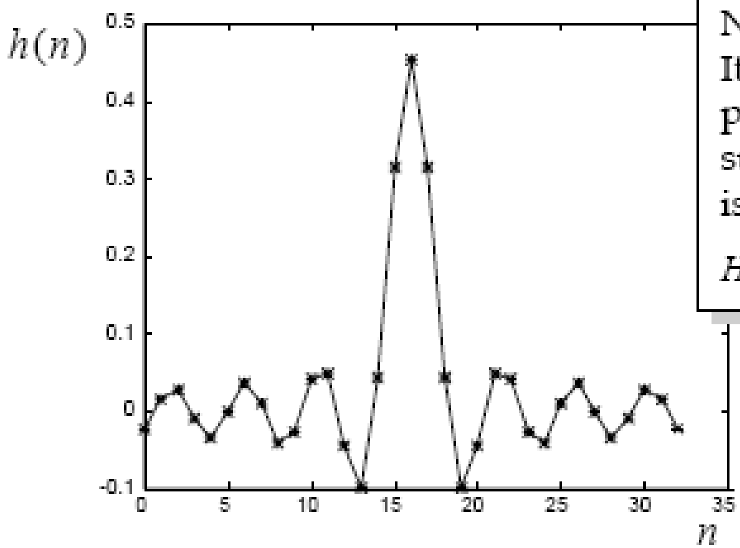


- For the ideal filter (from Fourier Series)

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{jn\omega}\, d\omega = \tfrac{1}{2}\frac{\sin(n\pi/2)}{n\pi/2} \qquad n = -\infty,\ldots,\infty$$

- For the truncated filter (from IDFT)

$$h(n) = \frac{1}{33}\sum_{k=0}^{32} H(k) e^{j\frac{2\pi}{N}nk}, \; n = 0,1,2,\ldots,32$$



Note that this result is causal. It is obtained using a linear phase assumption for the filter such that the delay of the filter is given by $(N-1)/2$ such that $H(e^{j\omega}) = \left|H(e^{j\omega})\right| \cdot e^{-j\omega(N-1)/2}$

Windowing

The truncation of the impulse response is equivalent to multiplication of the ideal (infinite) impulse response by a square window $w(n)$

$$h(n) = \begin{cases} h_d(n), & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

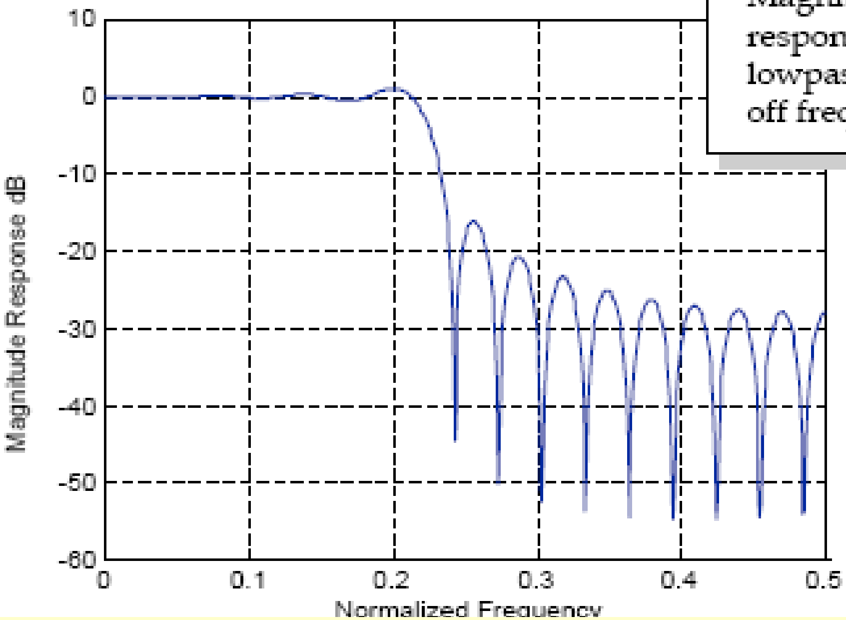
$$= h_d(n)w(n)$$

- Square window function

$$w(n) = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

◆ Effect of multiplying impulse response by window

- convolution of ideal frequency response with Fourier transform of window
- Fourier transform of square window is sinc
  - expect to see high side-lobes and ripples in the frequency response of the filter designed using square window



Magnitude of frequency response for the 33 taps lowpass filter with cut-off frequency of $\pi/2$

Other window functions

- Hamming window

$$w(n) = \begin{cases} 0.54 + 0.46\cos\left(\dfrac{n\pi}{I}\right), & -I \le n \le I \\ 0, & \text{otherwise} \end{cases}$$

- Hanning window

$$w(n) = \begin{cases} 0.5 + 0.5\cos\left(\dfrac{n\pi}{I}\right), & -I \le n \le I \\ 0, & \text{otherwise} \end{cases}$$

- (Several others)

- Use of raised cosine-type windows (Hamming or Hanning) gives better stopband attenuation but wider transition band

Filter magnitude responses for square, Hamming and Hanning windows